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Hadronic matrix elements from the lattice [a]

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Lattice matrix elements are briefly reviewed. In the quenched approximation f_B, B_B and B_K are now under good control. Experimental hints for $f_{D_S}^{\rm expt} > f_{D_S}^{\rm QQCD}$ are noted; precise determination of f_{D_S} from experiment as well as from the lattice is strongly advocated. Lattice calculations of the form factor for $B \to \pi \ell \nu$ at relatively large value of q^2 have made good progress and should be useful in conjunction with precise measurement of the differential spectra expected from B-factories. Recent attempt at $K \to \pi\pi$ using staggered quarks is briefly discussed; use of non-perturbative renormalization, improved actions and operators with staggered quarks is emphasized. Due to the good chiral behavior of domain wall quarks it would be useful to study $K \to \pi\pi$ with this discretization.

1 Introduction

Hadronic matrix elements are of crucial importance for constraining the parameters of the Standard Model (SM) in conjunction with experimental information. Lattice approach has already attained considerable success in handling B-parameters and decay constants and to a lesser degree semi-leptonic form factors. [1] Weak decays into purely hadronic final states, of which $K \to \pi\pi$ is the simplest example, are extremely problematic. [2] After almost a decade the topic is again getting some attention due in part to important developments with regard to chiral symmetry on the lattice. [3]

2 Heavy-light decay constants: effects of quenching. [1]

Lattice methods have made considerable progress in pinning down, fairly accurately, f_B and other heavy-light decay constants, in the quenched approxima-

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tion (see Table 1). Unfortunately, the effects of quenching could be substantial; current estimates place them around $\sim (20 \pm 10)\%$, seriously limiting the phenomenological applications. Since the past few years there is heightened activity to accurately ascertain the effects of quenching. For technical reasons, the dynamical simulations are with $N_f = 2$ i.e. with only two "light" sea-quarks rather than the three (u, d, s) in real life. Furthermore, the mass of the sea-quarks tends to be relatively heavy. The indications from these dynamical simulations is that $f_B^{\rm dynamical} > f_B^{\rm quenched}$.

Table 1: Heavy-light decay constants and their ratios. [1]

Quantity	Quenched ($N_f = 0$)	Partially Unquenched ($N_f = 2$)
f_B/MeV	170 ± 20	200 ± 30
f_{B_S}/MeV	190 ± 20	220 ± 30
f_D/MeV	205 ± 20	225 ± 30
f_{D_S}/MeV	225 ± 20	245 ± 30
f_{B_S}/f_B	$1.14 \pm .06$	$1.14 \pm .06$
f_{D_S}/f_D	$1.10 \pm .06$	$1.10 \pm .06$

3 Hints from Experiment: $f_{D_S}^{\rm expt} > f_{D_S}^{\rm QQCD}$?

Table 2 exhibits a compilation of the results for f_{D_S} from several experiments. Curiously, the central value of all but one experiment is above the value for f_{D_S} from quenched QCD simulations: $f_{D_S}^{\rm QQCD} = 225 \pm 20$ MeV. Since the errors in the existing experimental numbers are rather large we cannot draw strong conclusions; it seems plausible, nevertheless, that these experiments are also indicating that quenched QCD tends to underestimate the heavy-light pseudoscalar decay constants.

4 Precise studies of f_{D_S}

For experiment as well as for dynamical lattice simulations, a precise determination of $f_{B_D}(f_{B_S})$ is significantly more problematic than f_{D_S}; in fact experimentally direct determination of f_B is not of immediate reach. Therefore, it would be very useful if the experimental as well as the lattice determinations of f_{D_S} could be improved to 10–15% accuracy. A comparison between the two would then serve as a very useful guide for correcting $f_{B_d}(f_{B_S})$ from the lattice. In this regard the ratios f_{B_d}/f_{D_S} and f_{B_S}/f_{D_S} from the lattice would clearly be useful.

Table 2: Experimental determinations of f_{D_S}. [4]

Experiment	Mode	f_{D_S}/MeV
S. Aoki *et al.* (WA75)	$\mu^+\nu_\mu$	$238 \pm 47 \pm 21 \pm 48$
J.Z. Bai *et al.* (BES)	"	$430^{+150}_{-130} \pm 40$
K. Kodema *et al.* (E653)	"	$190 \pm 34 \pm 20 \pm 26$
M. Chada *et al.* (CLEO)	"	$280 \pm 19 \pm 28 \pm 34$
M. Acciari *et al.* (L3)	$\tau^+\nu_\tau$	$309 \pm 58 \pm 33 \pm 38$
F. Parodi *et al.* (DELPHI)	"	330 ± 95
(ALEPH)	inclusive	284 ± 62

5 Heavy-light B parameters

Lattice has had success with heavy-light B-parameters for a long time although calculations in the static approximation are still somewhat problematic. [1] Table 3 presents a brief summary. Recall that a precise value of the ratio $f_{B_S}(B_{B_S})^{1/2}/f_{B_d}(B_{B_d})^{1/2}$ is needed in accurately deducing V_{td}/V_{ts} once B_S-$\bar{B}_S$ oscillations get experimentally measured.

Table 3: Summary of heavy-light B-parameters. [1]

	Quenched	"Unquenched"
$B_{B_d}(m_b)$	.86(4)(8)	.86(4)(8)
B_{B_S}/B_{B_d}	1.00(1)(2)	1.00(1)(2)
$f_{B_d}(\hat{B}^{nlo}_{B_d})^{1/2}$	195 ± 25 MeV	230 ± 35
$\frac{f_{B_S}(B^{nlo}_{B_S})^{1/2}}{f_{B_d}(B^{nlo}_{B_d})^{1/2}}$	$1.14 \pm .06$	$1.14 \pm .07$

In view of the importance of the aforesaid ratio it may be useful to determine it also more directly from the SU(3) breaking ratio: [5]

$$\frac{M_{B_S}(\mu)}{M_{B_d}(\mu)} = \frac{\langle \bar{B}_{B_S}|(\bar{b}\gamma_\rho(1-\gamma_5)s)^2|B_{B_S}\rangle}{\langle \bar{B}_{B_d}|(\bar{b}\gamma_\rho(1-\gamma_5)d)^2|B_{B_d}\rangle}$$

Many of the systematic errors should cancel in this ratio. However, in the attempts that have been so far made with this direct method, [5,6] the errors are not yet small enough to make this method competitive with the traditional $f_B^2 B$ method.

6 Semi-leptonic form factors, $B \to \pi(\rho)\ell\nu$.

The large b-quark mass presents a difficult computational problem. The cleanest lattice simulations are for 'rest to rest' i.e. both the initial and final meson at rest. Then the large B mass forces q^2 (q = lepton 4-momentum $= p_B - p_\pi$) to be rather large. For some phenomenological applications the value of the form factor(s) are needed near $q^2 \sim 0$. This entails large extrapolations, introducing additional errors and model dependence. However, one important phenomenological application, namely deducing the mixing angle V_{ub} from experimental data, only requires precise knowledge of the form factor at one value of q^2. This should work so long as the experiment has enough data so that even the differential rate around that region of q^2 is accurately determined, which is anticipated to be feasible at the B-factories. In this regard two recent approaches are noteworthy. Both of these efforts avoid the use of large extrapolations in q^2 and focus instead on accurate predictions in a limited region of q^2.

UKQCD focussed on near the end-point or the zero-recoil region where the lattice data tends to be cleanest. Furthermore, heavy quark symmetry also provides useful scaling relations in this region. [8] Their result for the form factor f^+ as a function of q^2 is given below.

Table 4: $f^+(q^2)$ from UKQCD. [7]

$q^2(\mathrm{GeV}^2)$	16.7	18.1	19.5	20.9	22.3
$f^+(q^2)$	0.9^{+1+2}_{-2-1}	1.1^{+2+2}_{-2-1}	1.4^{+2+3}_{-2-1}	1.8^{+2+4}_{-2-1}	2.3^{+3+6}_{-3-1}

The FNAL group [9] is also focussing on an approach towards the semi-leptonic form factors for $B \to \pi\ell\nu$, $D \to \pi(K)\ell\nu$ suitable for accurate determinations of mixing-angles in conjunction with high statistics data samples anticipated from experiments. The key idea here again is to concentrate directly on the differential decay spectrum in an interval with $0.4 \lesssim \vec{p}_\pi/\mathrm{GeV} \lesssim 0.8$ thus avoiding the need for large extrapolation in q^2. The partial width over this interval can be computed on the lattice. From the experimental point of view this should have the advantage of using a range of q^2 wherein the decay rate is not small unlike near the end-point.

7 B_K

The kaon B-parameter, B_K, has been studied most extensively on the lattice. [2] Two important limitations that lattice simulations still need to adequately address are SU(3) breaking (the 'kaon' on the lattice is a pseudoscalar made of degenerate quarks with $m_{\rm pseudoscalar} \sim m_K$) and the quenched approximation. Both of these effects are expected to be rather small $\sim$ 5–10% and we get $\hat{B}_K = .85 \pm .13$. This number is based on results of various groups [2], amongst which the one from JLQCD is the most precise. [10] The error on the lattice number contains a guess-estimate of the SU(3) breaking and quenching errors. We note, in passing, that there are some preliminary indications that unquenching will increase [11] or decrease [12] B_K by just a few percent. For now, we have not changed the central value of B_K due to this effect; only the systematic errors are increased to reflect this possibility.

8 $K \to 2\pi$ Decays and ϵ'/ϵ.

It was realized long ago that chiral perturbation theory (ChPT) can be used to simplify the problem so that $\langle\pi\pi|Q|K\rangle$ can be obtained by computing on the lattice simpler entities: $\langle\pi|Q|K\rangle$ and $\langle{\rm vac}|Q|K\rangle$, where Q is a 4-quark operator. [13] The coefficients in this relation can be calculated using lowest order ChPT. Traditionally this strategy has been available for staggered fermions as they possess a remnant chiral symmetry. [14] Since Wilson fermions explicitly break chiral symmetry this approach cannot be used with this discretization. [15] The new development in this regard is that domain wall quarks (DWQ) [16] are found to be quite practical for simulating QCD and possess excellent chiral behavior. [17] Therefore, the $K \to \pi$ (and $K \to {\rm vac}$) method for dealing with $K \to 2\pi$ is amenable to this discretization as well. [3]

Final state interactions (FSI) are of course a serious limitation of the $K \to \pi$ method. However, it should still be very instructive to quantify how well this concrete approximation works. Actually direct $K \to 2\pi$ decays may also be amenable to the lattice due to an elegant application of the CPS symmetry. [18] The restrictions of the Maiani-Testa theorem [19] are bypassed here by working near the threshold. In any case the direct $K \to 2\pi$ methods are computationally extremely intensive and for now there is not much to report based on these methods.

The credit for an extensive study of the $K \to 2\pi$ in the $K \to \pi$ approach with staggered fermions goes to Pekurovsky and Kilcup (PK). [20,21] The work presents a first study of lattice spacing dependence, finite size effects as well as quenching effects. Unfortunately PK used lattice weak coupling perturbation theory [22] (LWCPT) to renormalize the operators and this seems to completely fail for staggered quarks in the case of LR operators such as Q_6 which are crucial for ϵ'/ϵ.

At $\beta = 6.0$, PK observe a significant enhancement of the $\Delta I = 1/2$ channel over the 3/2; however at $\beta = 6.2$, i.e. closer to the continuum limit (compared to $\beta = 6.0$), they find that the central value of the enhancement weakens appreciably. While the large errors at $\beta = 6.2$ do not allow a strong conclusion, more work is needed to unambiguously show that the $\Delta I = 1/2$ enhancement survives in the continuum limit.

PK's calculation of ϵ'/ϵ is seriously hampered as the one-loop LWCPT that they use for renormalization completely fails for Q_6. The perturbation theory corrections are several hundreds of percents showing extreme sensitivity to the renormalization scale as well as the quark mass (see Table 5). As PK themselves clearly emphasize their calculation of ϵ'/ϵ is extremely fragile and not at all reliable.

Table 5: $\langle Q_6 \rangle$ in arbitrary units with one-loop perturbative matching using two values of q^*; for comparison, the renormalized results ("bare") are also given (Table 2 from Ref. 20).

Quark Mass	0.01	0.02	0.03	0.04	0.05
$q^* = 1/a$	0.1 ± 1.2	-0.9 ± 0.4	-1.2 ± 0.2	-1.6 ± 0.3	-1.1 ± 0.2
$q^* = \pi/a$	-13.1 ± 1.8	-9.0 ± 0.5	-7.1 ± 0.3	-6.3 ± 0.5	-4.6 ± 0.5
Bare	-55.6 ± 5.0	-35.4 ± 1.5	-27.0 ± 0.9	-22.3 ± 1.4	-16.4 ± 1.5

It seems quite plausible that for the renormalization of the $\Delta S = 1$, 4-quark operators, as has also been known to some extent to be the case for quark bilinears, the breaking of flavor symmetry by the staggered approach is responsible for the failure of LWCPT. In any case, use of non-perturbative renormalization (NPR) [23] improved actions and/or operators with the staggered approach are highly desirable in the context of $K \to \pi\pi$ decays.

Domain wall quarks are extremely attractive as at the expense of introducing a fictitious 5^{th} dimension they preserve the full $\mathrm{SU}(N) \times \mathrm{SU}(N)$ chiral symmetries of the continuum theory in the limit of an infinite 5^{th} dimension. [16] Quenched QCD numerical simulations showed that in practice for $\beta \gtrsim 6.0$, 10–20 sites in the 5^{th} dimension may be sufficient to render very good chiral behavior. [17] Early numerical studies also seem to indicate that the discretization errors are effectively $0(a^2)$; if substantiated this improved scaling behavior may off-set the extra cost of the 5^{th} dimension. [17,24]

Calculation of $K \to 2\pi$ and ϵ'/ϵ in this method has been in progress for quite sometime. [3] With DWQ, considerable progress has been made in non-perturbative renormalization of quark bilinears, $\Delta S = 2$ and $\Delta S = 1$ Hamiltonians and so far the method seems promising. [25]

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